A longitudinal survey of Internet host reliability

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Abstract

An accurate estimate of host reliability is important for correct analysis of many fault-tolerance and replication mechanisms. In a previous study, we estimated host system reliability by querying a large number of hosts to find how long they had been functioning, estimating the mean timeto-failure (MTTF) and availability from those measures, and in turn deriving an estimate of the mean time-to-repair (MTTR). However, this approach had a bias towards more reliable hosts that could result in overestimating MTTR and underestimating availability. To address this bias we have conducted a second experiment, using a fault-tolerant replicated monitoring tool. This tool directly measures TTF, TTR, and availability by polling many sites frequently from several locations. We find that these more accurate results generally confirm and improve our earlier estimates, particularly for TTR. We also find that failure and repair are unlikely to follow Poisson processes.

1 Introduction

Accurate analyses of distributed fault-tolerance, caching, and replication mechanisms depend on an accurate model of the reliability of the systems that make them up. The overall reliability of a replication protocol, for example, depends on the probability that some fraction of the replica sites are functioning when data must be read or written.

There are several important measures used to quantify system reliability, including *time-to-failure* (TTF), *time-torepair* (TTR), *availability*, and *reliability*. Throughout this study, "failure" is defined in a distributed-environment sense: that is, as an inability to access a host within a specified time. The term encompasses both hardware and software faults attributable to the host, and can include power failures and scheduled downtime. It can also be caused by off-site communications failures, ranging from temporary routing failures to problems with the physical communications links. We have not attempted to characterize the causes of failure, though it seems that most failures are brief and are probably caused by software faults or voluntary reboots. Richard Golding

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Time-to-failure and *time-to-repair* are the distributions of how long a system is available for use, and how long it takes to bring the system back to normal operation after a failure has occurred. These distributions are often summarized by their means (MTTF and MTTR).

Many studies have assumed that failure and repair followed Poisson processes, that is, that the distributions for TTF and TTR were exponential. This assumption is often made more for analytic simplicity than out of a conviction that it is the best model of reality. For example, some analyses of replication protocols using Markov models [Pâris86, Long89] depend on that assumption. We investigate the accuracy of this assumption in §3.1.2 and §3.2.2.

Availability is the fraction of time a system is functioning. More precisely, it is the stationary probability of the system being in a state where it can be accessed. For a replication protocol, for example, it gives the probability that a replica site will be functioning when a client starts executing a read or write protocol. Availability can also be used in conjunction with MTTF to derive an estimate of MTTR.

Reliability is the (non-stationary) probability that a system will remain constantly available over a fixed time period. Consider a system that functions for one second, then fails, but recovers in a small number of milliseconds. This system would have a high availability, but would not be useful for applications like process control that must remain continuously functioning for extended periods. Reliability is a more appropriate measure for these applications because it includes duration.

There have been few analyses of host system reliability published, and most of those have been for specific systems. Recent studies include analyses of Tandem systems [Gray85, Gray90] and the IBM/XA system [Mourad85]. It is certain that most companies perform reliability studies of their products internally.

We have measured the reliability of a wide variety of host systems connected to the Internet. For this study, we monitored nearly 1 200 hosts for an extended period, polling them to determine how long they were available. The emphasis on a heterogeneous set of hosts, selected so that its composition should be similar to the overall population of Internet hosts, makes our results more generally applicable than studies specific to one type of system.

We conducted a similar study four years ago [Long91], but the method we used to estimate time-to-failure and timeto-repair was biased. Estimates of MTTF were derived by querying systems for their up-time. This was the best information available from the host system, since it is not generally in a position to give its failure time as its dying gasp. As a result, there was a bias towards more reliable hosts which means that the estimate of MTTF may be larger than the true value.

Our new study uses direct measurement of TTF and TTR, rather than an estimate. We measured these distributions by polling each selected host every few minutes. The resulting distributions should reflect all but the shortest periods of failure. Instead of estimating parameters such as MTTF based on a large sample with an unknown distribution, we recorded the actual events (with an epsilon error). Since the quantities are being directly measured, questions such as the governing distribution are less important.

The current study used a distributed, fault-tolerant measurement system (the *Tattler* [Long92]) to reduce bias from the measurement approach. The tattler system consists of replicated monitoring sites placed at strategic locations around the Internet. Individual tattlers were placed to minimize the amount of shared network so that a failure of a router or a link was unlikely to disable more than one monitor. The tattlers replicated their measurements using a weak-consistency group communication mechanism so that even the permanent failure of some monitors would not cause a significant loss of information.

In the sections that follow, we present our experimental method, followed by the results we obtained and our analysis.

2 Experimental method

We began the study by selecting a large number of candidate hosts for measurement, and eliminating unsuitable ones. The Tattler system then monitored these hosts for an extended period to measure their TTF and TTR. In this section we detail how these steps were accomplished.

2.1 Selecting hosts

Our first step was to find a list of at least a thousand hosts that could respond to our polling (using RPC calls to rpc.statd, which is common on systems using NFS). We wanted a method that would probably yield a set of hosts statistically similar to the overall population of hosts on the Internet.

We began by compiling a list of all visible host names on the Internet. We used the Census tool [Ganatra92], which queries the top-level DNS servers for the names of the hosts at each site and for secondary domain servers, then recursively applies the process to the entire visible Internet DNS name-tree. This resulted in almost three million hosts nearly ten times the figure we reported four years ago.

This approach does not consider all the hosts connected to the Internet. Some installations, particularly corporations, choose to shield their internal network behind a gateway; those subnetworks are thus invisible to the network at large.

From the census results we chose 15 000 hosts at random. This list was then filtered to ensure that the hosts actually existed, could respond to the poll, and that their administrators would not mind the poll. In the end we obtained a list of 1 170 hosts.

These hosts were uniformly distributed over the name space, and all responded to our RPC polls. Geographic locations with more hosts had a stronger representation than those with fewer. Limiting ourselves to systems responding to the rpc.statd protocol biased the list, but we believe that the typical workstation on the Internet—the kind of system that is of most interest to us—is likely to include this protocol.

In our previous study we grouped hosts by type, based on information reported in the name service's host information (HINFO) records. This has proven impossible in the newer study. We found that in general these records are not as well maintained now as they were four years ago, so that in most cases we cannot determine what kind of host we are polling. Many sites no longer provide HINFO records, and many of the ones that were provided did not include enough information to determine system models.

2.2 Measuring host systems

There were two ways we could have measured the systems. In our first study, we queried each site to determine the time since its last initialization and used this to estimate MTTF, and separately estimated overall availability. In this study, we chose to take direct measurements of the distributions of TTF and TTR by repeated polling each site for an extended period.

The first method has the advantage of requiring only a single query for each host, but the results are not as accurate as those of the second. Randomly sampling the length of time since the last system initialization, which we used in the first study, is distinct from sampling the length of time between initialization and failure. Sampling system up-time reports results in a skewed set of data, as hosts which have been up the longest are more likely to be polled. Analysis of the data must accommodate this effect.

In this study, on the other hand, we directly measured TTF and TTR by polling each of the selected hosts frequently. We used an exponential distribution of times between polls, with a mean of ten minutes, both for statistical purposes and to avoid synchronous behavior where multiple tattlers poll the same host at once.

The Tattler system, the monitoring tool we built, maintained a data base of *available epochs* for each host being monitored. Each epoch represented one period when the host was known to be functioning, and was stored as a tuple <host, boot time, last sample time>. The intervals between the available epochs were treated as periods when the host was unavailable.

Each poll returned either the time since the host was last initialized, or a failure. The durations were merged into the list of available epochs for the host, extending an epoch if they had the same boot time.

At the end of the experiment, we extracted the list of available epochs for each measured host and computed the distributions for TTF and TTR. Our analysis then derived availability from these values.

The accuracy of our measurements depended on each machine maintaining an accurate record of its boot time and current time of day. We observed some systems that appeared not to maintain this information, resulting in negative or multi-decade uptimes; we removed these outliers before analyzing the data.

The measurements were also sensitive to the polling frequency. If a machine failed after less than ten minutes, the Tattler might well miss the period the host was available, though it would detect very short downtimes using the boot time in each sample.

2.3 The Tattler system

We built the Tattler as a distributed, fault-tolerant system. The Tattler is composed of a number of replicated monitors, the individual *tattlers*, at geographically dispersed sites, as shown in Figure 1. These monitors are called tattlers since they periodically inquire about other hosts and then "tattle" to each other about what they learn. In practice we used six tattlers: four at the University of California,

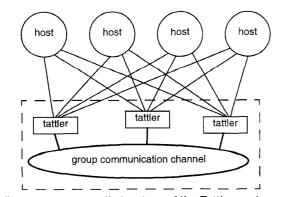


Figure 1: The overall structure of the Tattler system

Santa Cruz; one at San Diego State University; and one at the Georgia Institute of Technology.

There are several advantages to replicating the tattlers around the network. First, it provides a fault-tolerant method for monitoring hosts. All but one of the tattlers can fail and the set of hosts can still be monitored (albeit in a degraded mode). It also provides a way of mitigating the effect of transient network failures. When monitoring hosts from a single point, the failure of one router can prevent any host from being polled. When several relatively independent polling daemons work together, it will be very unlikely that a total failure can occur. Second, because the tattlers are distributed they can perform many more queries than a single polling program with six tattlers, each tattler polled each host on average only once an hour to achieve an overall mean time between polls of ten minutes. While a single polling program would create roughly the same message traffic, it would concentrate all the traffic onto a small number of network links from the monitoring host to the Internet backbone networks. The parallelism of using multiple nodes also decreases the load on them, so we could use ordinary workstations without disturbing their users. A non-replicated monitor would also take significantly longer to complete its task since it would have to poll for a longer period to make up for data lost due to failures.

Each tattler maintained a copy of the list of hosts to poll, and of the data base of available epochs for each host. The list of hosts was a sequence of tuples <host, poll method, poll interval>. The data base contained a sequence of tuples, one per available epoch for each host being monitored, of the form <host, boot time, sample time>.

The tattler replicas were coordinated using a weak consistency group communication protocol [Golding92]. This protocol provides operations for new replicas to join the group, sending the new replica a copy of the data base in the process; for leaving the group, when a tattler is to be shut down; and for propagating metadata and data base changes. The tattlers did not communicate in real time; instead, they proceeded independently and periodically merged their data bases. This allowed the system to continue functioning when individual tattlers were temporarily unavailable or the network had partitioned—common events for a system built on the Internet.

An individual tattler was composed of several components: a tattler daemon, a data base daemon, and a polling daemon, as shown in Figure 2. The tattler daemon coordinated the other daemons, and was responsible for managing the consistency of the replicated data base through the group communication channel. The data base daemon provided stable storage for sample observations (from the polling daemon), and metadata from the tattler daemon.

The polling daemon produced sample observations. It took samples at a specified rate, and could be requested to start

or stop sampling. For this study, it used exponentially distributed random intervals with a mean of one hour.

The system also provided a user interface for controlling individual tattlers. It allowed hosts to be added and deleted from the monitoring list, and allowed a user to suspend monitoring of certain hosts. It could inform a tattler that it should shut down and leave the process group. New tattlers could be added equally easily. The user interface contacted a single tattler—preferably the closest—to perform all of these operations, and the group communication protocols ensured that the operation was eventually known by all tattlers.

3 Results

This study was conducted over a seven-month period. The study is continuing, and as more time passes the accuracy of our results will improve.

3.1 Time-to-failure

During the seven-month experimental period, we observed 13 250 intervals when systems were available.

There are two ways to analyze these periods: to aggregate all hosts together, or to separate them. The results are notably different because the aggregated measures are heavily influenced by unreliable hosts that contributed many short available epochs. The aggregated results are useful if one is trying to characterize the "average" host—perhaps as might be seen as a client of a wide-area service. Different makes of host, however, are likely to fail in different ways and indeed analysis of our data indicate that the hosts are not homogeneous—so we also present unaggregated statistics.

3.1.1 Aggregated TTF

The average (aggregated) duration of all available periods was 12.99 days, or 311.8 hours. Table 1 summarizes these measurements.

The distribution is rather skewed, with a few very long intervals. Figure 3 shows the density.

These values are close to what system administrators would expect, according to anecdotal reports we have gathered. The values differ greatly from the MTTF values

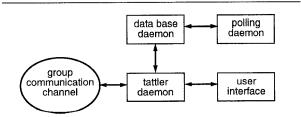


Figure 2: The structure of an individual tattler replica

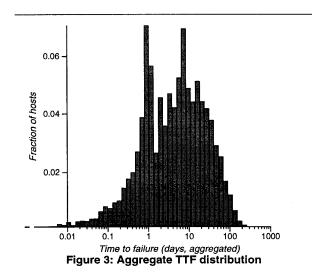
Table 1: Measured aggregate TTF

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mean 12.99 days (311.8 hours)
           \pm 0.15 (50% confidence)
           \pm 0.42 (95% confidence)
           \pm 0.56 (99% confidence)
median 4.080 days
        13 250 intervals
     n
        24.92
    σ
        0.000428
   min
  25\%
        0.8986
  50%
        4.080
  75%
        14.12
        550.4
  max
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reported by manufacturers because we are using a different definition of failure: manufacturers are generally concerned with permanent hardware failures, while we are concerned with the ability to communicate with a service.

The number reported is also much shorter than the time between operating system or hardware "failures". Manually shutting down a machine—for software maintenance or to conserve power, for example—make the system unavailable according to our definition. Many people reboot their systems nightly or weekly, contributing many short intervals. The spike at one day in Figure 3 bears this out.

This difference may also account for another difference between our observation and common wisdom: that failure rates follow the "bathtub curve" so that failures are likely soon after a repair has been completed. Figure 3 does not exhibit this behavior. This would be expected, however, if many system shutdowns are planned reboots that do not involve changes to the system's configuration.



3.1.2 Is failure a Poisson process?

In our previous study, we found evidence that the timesince-initialization values we measured were not exponentially distributed, and argued that if TTF were exponential, time-since-initialization must be as well indicating that TTF most likely does not follow a Poisson process. In this section, we apply the same test statistic to the data we collected in this study.

We use a test statistic based on the parametric family of distributions with linear failure rate density, which has been shown to be applicable to a large class of nonparametric distributions as well, and has been shown to be applicable to machine behavior [Doksum84]. This test does not depend on advance knowledge of the mean of the proposed governing distribution. For n samples t_1 through t_n with mean t, the test statistic is given by:

$$T = \frac{1}{\sqrt{n}} \sum_{i=1}^{n} \left[1 - \frac{1}{2} \left(\frac{t_i}{t} \right)^2 \right]$$

If the null hypothesis H_0 that the samples come from a single exponential distribution is true, the test statistic *T* has a standard normal distribution when the sample size *n* is large. Thus the null hypothesis H_0 can be rejected at a specified level of significance when the value of the equivalent formula

$$T = \frac{1}{2}\sqrt{n} \left[1 - \frac{\hat{\sigma}^2}{t^2} \right]$$

is large, where $\hat{\sigma}^2$ is the sample variance.

The test statistic T can be also used in testing the null hypothesis that the samples come from a population with a linear failure-rate density as well as a population with a nondecreasing failure-rate average. For these cases, the null hypothesis can be rejected for extreme values of the test statistic T; the significance probability is calculated from the standard normal distribution.

No matter how large the sample size, no amount of testing can assure that a population distribution is exponential. By contrast, the test statistic T can quantify the prohibitively small probability that certain samples were derived from an exponential population distribution.

We calculated T for the TTF values for each host, and found -3280, which gives a vanishingly small probability that TTF is exponentially distributed.

3.1.3 Unaggregated TTF

Rather than aggregating all hosts, we can also consider them separately, examining the distribution of mean TTF for the host population we followed. We would expect this measure to be less influenced by unreliable hosts than the aggregated one, and to help show differences in the failure processes of different hosts.

The average mean duration of all hosts was 29.39 days, or 705.4 hours—significantly longer than the aggregated measure, as expected. Table 2 summarizes these measurements.

Table 2:	Measured	MTTF
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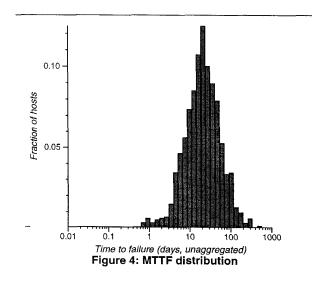
mean	29.39 days (705.4 hours)
	± 0.77 (50% confidence)
	± 2.3 (95% confidence)
	± 3.0 (99% confidence)
median	18.12 days
n	1139 hosts
σ	38.76
min	0.1419
25%	9.406
50%	18.12
75%	34.58
max	546.9

The distribution is also skewed, with a few very long intervals, as shown in Figure 4.

3.1.4 Comparison with previous study

In our previous study, we observed MTTF values ranging from 15.85 to 20.14 days, depending on the kind of system, as compared to our observation of 24.92 days (aggregated) and 29.39 days (unaggregated) in this study. Those systems with a large population tended to be in the upper portion of that range, with MTTF between 17.96 to 20.14 days.

The samples in our previous study were biased to overrepresent more reliable systems, which increased the time-tofailure measure. These new results confirm the bias in the



earlier study, and encourage our belief that we have remedied it.

3.2 Time-to-repair

The measured overall mean aggregated TTR was 2:018 days (48.43 hours), with 12 053 epochs when systems were unavailable. Table 3 summarizes the results.

Once again the distribution is strongly skewed, with the median time (just over seven hours) much shorter than the mean. Figure 5 shows the distribution.

Table 3:	Measured	aggregate	TTR
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mean	2.018 days
	± 0.097 (50% confidence)
	± 0.281 (95% confidence)
	±0.369 (99% confidence)
median	0.2941 days
n	12 053 intervals
σ	15.73
min	0.000660
25%	0.09741
50%	0.2941
75%	1.259
max	1449

3.2.1 Unaggregated MTTR

We can also consider the distribution of MTTR over the various hosts. The measured overall mean MTTR was 3.880 days (93.11 hours) for 1081 hosts. Table 4 summarizes the results.

Once again the distribution is strongly skewed, with the median time (just over 36 hours) much shorter than the mean. Figure 6 shows the distribution.

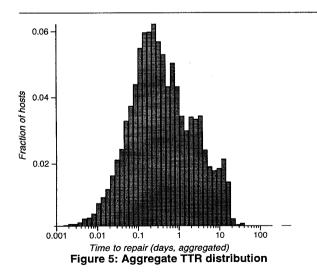


Table 4: Measured MTTR

mean	3.880 days
	± 0.265 (50% confidence)
	± 0.771 (95% confidence)
	± 1.01 (99% confidence)
median	1.508 days
n	1081 hosts
σ	12.93
min	0.002662
25%	0.8598
50%	1.508
75%	2.552
max	147.2
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3.2.2 Is repair a Poisson process?

As with failure, repair is often assumed to follow a Poisson process. We applied the same test statistic to our measured repair times, which yielded T = -154.4. Once again, the chance that repair is exponential is vanishingly small.

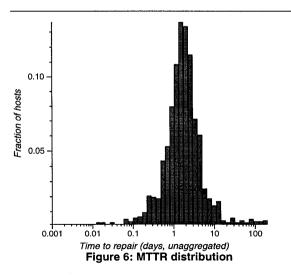
3.2.3 Comparison with previous study

In our previous study, we estimated MTTR values between 1.86 and 2.96 days for the most common kinds of hosts, and with few exceptions the MTTR estimates for other models were notably longer than our current measurements.

How could significant errors have been introduced into our previous estimates of MTTR? In that study we derived MTTR from the MTTF and availability A by:

$$MTTR = \frac{MTTF(1-A)}{A}$$

This is most easily derived from two state birth-death process, but by using results from the theory of renewal processes this



result can be shown to be independent of the distributions of MTTF and A [Trivedi82].

The MTTR estimate is linearly dependent on MTTF. If the MTTF were overestimated, the estimated MTTR would be proportionally longer than the actual. Also, if the confidence interval for the availability of the MTTF is large, then the estimated MTTR can deviate significantly from the true value.

An accurate MTTR also depended on the accuracy of the availability measure. Consider the rate at which the value of MTTR changes when an error is made in the estimate of A:

$$\frac{\partial}{\partial A} \frac{MTTF(1-A)}{A} = \frac{-MTTF}{A^2}$$

The error introduced into the estimate of MTTR is quadratic in the error in the estimate of A. As we will see, the availability measure from the earlier study was flawed.

3.3 Availability

In this study, we computed availability as the ratio of the time we observed that a host was available to the total time, both failed and functioning. We computed an overall availability of 0.8783 for a population of 1 134 hosts, or a mean of 46 days unavailable per year. Table 5 summarizes.

Table 5: Computed availability

mean	0.8783
	± 0.00297 (50% confidence)
	± 0.00864 (95% confidence)
	±0.0114 (99% confidence)
п	1134 hosts
median	0.9203

Figure 7 shows a histogram of the distribution over the hosts. Again, the distribution is highly skewed. Only a small fraction of the hosts (11.7%) were available less than 75% of the time.

Many of the hosts (242, or 5%) have availability of 1.0. These hosts have not failed in the time we have been monitoring them. We expect that as we continue to monitor them we will be able to better characterize these highlyreliable hosts.

The time that these systems are unavailable—several days per year—is very different from anecdotal accounts of behavior of dedicated server systems. In general we believe that this is because we are monitoring the average host on the Internet, which is likely to be a workstation-class machine for a single user or for a small workgroup. These users do not generally invest in uninterruptible power supplies, redundant processors, or other special mechanisms for ultra-highly reliable systems.

3.3.1 Comparison with previous study

In the previous study we reported much lower availability for most kinds of systems. This in turn led to substantially different results for MTTR. We measured availability by polling a large list of host names to determine which names corresponded to real systems, then polling again two months later to measure what fraction were reachable.

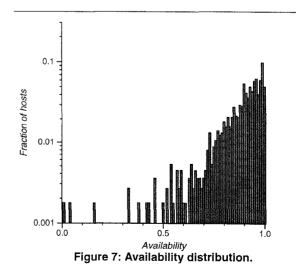
That method suffered from two problems. First, it overrepresented reliable systems, since the poll actually measured the conditional probability of being able to reach the host a second time, and the probabilities of the polls being successful were not independent. Second, it did not differentiate between the reliability of the network and the reliability of the host. A transient network failure that lost the query packets would be indistinguishable from a host failure, and packet loss is likely when network segments become congested.

4 Summary

We have performed a longitudinal measurement study of the TTF and TTR of a sample of hosts connected to the Internet. We collected data from almost 1 200 hosts, using only data that could be obtained via the Internet with no special privileges or added monitoring facilities.

We chose to directly measure TTF and TTR, rather than trying to estimate them from other measures. We did this by polling each host regularly to determine how long it had been functioning. We ran the monitor for three months to obtain long-term measures of TTF and TTR. We used these measures to compute overall availability.

The results of the previous study reflected a bias toward more reliable hosts that needed addressing. Our use of direct



measurement improves upon the earlier results. We now believe that we have eliminated all significant sources of bias in the measurements, and in so doing have presented the most accurate statistic on host reliability available.

In order to accomplish this task, we constructed the Tattler, a fault-tolerant distributed monitor that continuously polled hosts over the Internet. Its distributed nature allowed us to avoid putting too great a load on any particular host or on any particular segment of the Internet. By dispersing the tattlers geographically, we were able to minimize the effects of network failures, which biased the previous availability estimate.

From our TTF measurements, it appears that many systems stay up for about one week. The data suggest daily and weekly patterns, where systems are unavailable for a short period each day or week. For both TTF and TTR, the mean and median are significantly different, suggesting that only a few systems stay up a long time, but that most are unavailable for only a short time.

The TTF and TTR distributions are clearly not exponential. We believe that care needs to be taken in future reliability analyses to ensure that assumptions about exponential behavior do not cause problems. As more data are collected we hope to be able to better explore the distributions that these take.

We intend to continue the monitoring experiment for the foreseeable future, and including at least one new cohort of systems, focussing on server hosts. Those interested in participating in this study, by running tattlers on their systems, should contact the authors.

Acknowledgments

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